Radar Systems - Range Equation

Radar range equation is useful to know the range of the target **theoretically**. In this chapter, we will discuss the standard form of Radar range equation and then will discuss about the two modified forms of Radar range equation.

We will get those modified forms of Radar range equation from the standard form of Radar range equation. Now, let us discuss about the derivation of the standard form of Radar range equation.

Derivation of Radar Range Equation

The standard form of Radar range equation is also called as simple form of Radar range equation. Now, let us derive the standard form of Radar range equation.

We know that **power density** is nothing but the ratio of power and area. So, the power density, P_{di} at a distance, R from the Radar can be mathematically represented as −

$$P_{di} = \frac{P_t}{4\pi R^2} \quad Equation\ 1$$

Where,

P_t is the amount of power transmitted by the Radar transmitter

The above power density is valid for an isotropic Antenna. In general, Radars use directional Antennas. Therefore, the power density, P_{dd} due to directional Antenna will be −

$$P_{dd} = \frac{P_t G}{4\pi R^2} \quad Equation\ 2$$

Target radiates the power in different directions from the received input power. The amount of power, which is reflected back towards the Radar depends on its cross section. So, the power density P_{de} of echo signal at Radar can be mathematically represented as −

$$P_{de} = P_{dd} \left(\frac{\sigma}{4\pi R^2}\right) \quad Equation\ 3$$

Substitute, Equation 2 in Equation 3.

$$P_{de} = \left(\frac{P_t G}{4\pi R^2}\right)\left(\frac{\sigma}{4\pi R^2}\right) \quad Equation\ 4$$

The amount of **power, P_r received** by the Radar depends on the effective aperture, A_e of the receiving Antenna.

$$P_r = P_{de} A_e \quad Equation\ 5$$

Substitute, Equation 4 in Equation 5.

$$P_r = \left(\frac{P_t G}{4\pi R^2}\right)\left(\frac{\sigma}{4\pi R^2}\right) A_e$$

$$\Rightarrow P_r = \frac{P_t G \sigma A_e}{(4\pi)^2 R^4}$$

$$\Rightarrow R^4 = \frac{P_t G \sigma A_e}{(4\pi)^2 P_r}$$

$$\Rightarrow R = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 P_r}\right]^{1/4} \quad Equation\ 6$$

Standard Form of Radar Range Equation

If the echo signal is having the power less than the power of the minimum detectable signal, then Radar cannot detect the target since it is beyond the maximum limit of the Radar's range.

Therefore, we can say that the range of the target is said to be maximum range when the received echo signal is having the power equal to that of minimum detectable signal. We will get the following equation, by substituting $R = R_{Max}$ and $P_r = S_{min}$ in Equation 6.

$$R_{Max} = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}}\right]^{1/4} \quad Equation\ 7$$

Equation 7 represents the **standard form** of Radar range equation. By using the above equation, we can find the maximum range of the target.

Modified Forms of Radar Range Equation

We know the following relation between the Gain of directional Antenna, G and effective aperture, A_e.

$$G = \frac{4\pi A_e}{\lambda^2} \quad Equation\ 8$$

Substitute, Equation 8 in Equation 7.

$$R_{Max} = \left[\frac{P_t \sigma A_e}{(4\pi)^2 S_{min}} \left(\frac{4\pi A_e}{\lambda^2}\right)\right]^{1/4}$$

$$\Rightarrow R_{Max} = \left[\frac{P_t G \sigma {A_e}^2}{4\pi \lambda^2 S_{min}}\right]^{1/4} \quad Equation\ 9$$

Equation 9 represents the **modified form** of Radar range equation. By using the above equation, we can find the maximum range of the target.

We will get the following relation between effective aperture, A_e and the Gain of directional Antenna, G from Equation 8.

$$A_e = \frac{G\lambda^2}{4\pi} \quad Equation\ 10$$

Substitute, Equation 10 in Equation 7.

$$R_{Max} = \left[\frac{P_t G \sigma}{(4\pi)^2 S_{min}} (\frac{G\lambda^2}{4\pi})\right]^{1/4}$$

$$\Rightarrow R_{Max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^2 S_{min}}\right]^{1/4} \quad Equation\ 11$$

Equation 11 represents **another modified form** of Radar range equation. By using the above equation, we can find the maximum range of the target.

Note − Based on the given data, we can find the maximum range of the target by using one of these three equations namely

- Equation 7
- Equation 9
- Equation 11

Example Problems

In previous section, we got the standard and modified forms of the Radar range equation. Now, let us solve a few problems by using those equations.

Problem 1

Calculate the **maximum range of Radar** for the following specifications −

- Peak power transmitted by the Radar, $P_t = 250KW$
- Gain of transmitting Antenna, $G = 4000$
- Effective aperture of the receiving Antenna, $A_e = 4\ m^2$
- Radar cross section of the target, $\sigma = 25\ m^2$
- Power of minimum detectable signal, $S_{min} = 10^{-12} W$

Solution

We can use the following **standard form** of Radar range equation in order to calculate the maximum range of Radar for given specifications.

$$R_{Max} = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}}\right]^{1/4}$$

Substitute all the given parameters in above equation.

$$R_{Max} = \left[\frac{\left(250 \times 10^3\right)(4000)(25)(4)}{(4\pi)^2 \left(10^{-12}\right)}\right]^{1/4}$$

$$\Rightarrow R_{Max} = 158\ KM$$

Therefore, the **maximum range of Radar** for given specifications is $158\ KM$.

Problem 2

Calculate the **maximum range of Radar** for the following specifications.

- Operating frequency, $f = 10GHZ$
- Peak power transmitted by the Radar, $P_t = 400KW$
- Effective aperture of the receiving Antenna, $A_e = 5\ m^2$
- Radar cross section of the target, $\sigma = 30\ m^2$
- Power of minimum detectable signal, $S_{min} = 10^{-10} W$

Solution

We know the following formula for **operating wavelength**, λ in terms of operating frequency, f.

$$\lambda = \frac{C}{f}$$

Substitute, $C = 3 \times 10^8 m/sec$ and $f = 10GHZ$ in above equation.

$$\lambda = \frac{3 \times 10^8}{10 \times 10^9}$$

$$\Rightarrow \lambda = 0.03m$$

So, the **operating wavelength**,λ is equal to $0.03m$, when the operating frequency, f is $10GHZ$.

We can use the following **modified form** of Radar range equation in order to calculate the maximum range of Radar for given specifications.

$$R_{Max} = \left[\frac{P_t \sigma {A_e}^2}{4\pi\lambda^2 S_{min}}\right]^{1/4}$$

Substitute, the given parameters in the above equation.

$$R_{Max} = \left[\frac{\left(400 \times 10^3\right)(30)\left(5^2\right)}{4\pi(0.003)^2(10)^{-10}}\right]^{1/4}$$

$$\Rightarrow R_{Max} = 128KM$$

Therefore, the **maximum range of Radar** for given specifications is $128\ KM$.